Time saving method based on angular quantities applied to an internal multiple attenuation algorithm: fundamental concept, development and numerical analysis.

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SUMMARY

The Inverse Scattering Series (ISS) is a direct inversion method for a multidimensional acoustic, elastic and anelastic earth. It communicates that all inversion processing goals are able to be achieved directly and without any subsurface information. This task is reached through a task-specific subseries of the ISS. Using primaries in the data as subevents of the first-order internal multiples, the leading-order attenuator can predict the time of all the first-order internal multiples and is able to attenuate them.

However, the ISS internal multiple attenuation algorithm can be a computationally demanding method specially in a complex earth. By using an approach that is based on two angular quantities and that was proposed in Terenghi et al. (2012), the cost of the algorithm can be controlled. The idea is to use the two angles as key-control parameters, by limiting their variation, to disregard some calculated contributions of the algorithm that are negligible. Moreover, the range of integration can be chosen as a compromise of the required degree of accuracy and the computational time saving.

This time-saving approach is presented in this paper and applied to the ISS internal multiple attenuation algorithm. Through a numerical analysis, the relationship between accuracy and performance is examined and discussed.

INTRODUCTION

Araújo et al. (1994) and Weglein et al. (1997) have proposed the ISS internal multiple attenuation algorithm. It is a leading order contribution towards the elimination of first order internal multiples. The algorithm is based on the construction of an internal multiple attenuator coming from a subseries of the ISS. It has received positive attention for stand-alone capability for attenuating first-order internal multiples in marine and off-shore plays.

Terenghi et al. (2012) introduced two angular quantities that can be used as a key-control of the computational cost of the ISS leading order internal multiple attenuation algorithm. The two angles, the dip angle and the incidence angle, are related to the wavefield variables in the f-k domain. Therefore, control of this angles can be key to our ability to control the time loop of the algorithm. In this paper, we will discuss how the computational cost can relate to the accuracy to the internal multiples prediction. In other words, is it possible to reduce the computational time of the ISS internal multiple attenuation algorithm without affecting its efficiency?

In the first part of this paper, a description of the internal multiple attenuation algorithm will be provided. Then, the angle constraints method will be develloped and applied to the ISS internal multiple attenuation algorithm. Finally, a numerical analysis will be shown in order to discuss the relation between the accuracy and efficiency of the algorithm, and this key-control parameters.

THE INVERSE SCATTERING SERIES INTERNAL MULTIPLE ATTENUATION ALGORITHM

In seismic processing, many methods make assumptions and require subsurface information. However sometimes these assumptions could be difficult or impossible to satisfy in a complex world. The Inverse Scattering Series states that all processing objectives can be achieved directly and without any subsurface information.

The Inverse Scattering Series is based on scattering theory which is a form of a perturbation analysis. It describes how a scattered wavefield (the difference between the actual wavefield and the reference wavefield) relates to the perturbation (the difference between the actual medium and the reference medium).

The forward scattering series construction starts with the differential equations governing wave propagation in the media:

$$LG = \delta(r - r_s), \tag{1}$$

$$L_0 G_0 = \delta(r - r_s). \tag{2}$$

With L and L_0 the actual and the reference differential operators. And G and G_0 are the actual and reference Green's functions. We define the scattered field as $\psi_s = G - G_0$ and the pertubation as $V = L_0 - L$.

The Lippmann-Schwinger equation relates G, G_0 and V:

$$G = G_0 + G_0 V G \tag{3}$$

Substituing iteratively the Lippmann-Schwinger equation into itself gives the forward scattering series:

$$\begin{aligned} \psi_s &= G_0 V G_0 + G_0 V G_0 V G_0 + G_0 V G_0 V G_0 V G_0 + \ldots \\ &= (\psi_1) + (\psi_2) + (\psi_3) + \ldots, \end{aligned} \tag{4}$$

Where, (ψ_n) is the portion of the scattered wavefield that is the nth order in V. The measured values of ψ_s are the data D.

The pertubation V can also be expanded as a series,

$$V = V_1 + V_2 + V_3 + \ldots \tag{5}$$

Substituing V into the forward scattering series, and evaluating the scattered field on the measurement surface results in the inverse scattering series:

$$(\psi_s)_m = (G_0 V_1 G_0)_m \tag{6}$$

$$0 = (G_0 V_2 G_0)_m + (G_0 V_1 G_0 V_1 G_0)_m \tag{7}$$

$$\begin{aligned} 0 = {} & (G_0 V_3 G_0)_m + (G_0 V_2 G_0 V_1 G_0)_m + \\ & (G_0 V_1 G_0 V_2 G_0)_m + (G_0 V_1 G_0 V_1 G_0 V_1 G_0)_m \end{aligned} \tag{8}$$

$$\ldots$$

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The Inverse Scattering Series internal multiple attenuation concept is based on the analogy between the forward and the inverse series. The forward series could generate primaries and internal multiples through the action of G_0 on the perturbation V, while, the inverse series can achieve a full inversion of V by using G_0 and the measured data. The way that G_0 acts on the pertubation to construst the internal multiples suggests the way to remove them. In the forward series the first-order internal multiples have their leading-order contribution from the third term: $G_0VG_0VG_0VG_0$. This suggests that the leading-order attenuator of internal multiples can be find in the third term in the inverse series equation (8). In Weglein et al. (1997), a subseries that attenuates internal multiples was identified and separated from the entire inverse scattering series.

The ISS internal multiple attenuation algorithm is a subseries of the inverse scattering series. The algorithm begins with the input data $D(k_g,k_s,\omega)$ which is the data in the ω temporal frequency deghosted and with free-surface multiple removed. This means that they are only primaries and internal multiples in the data. With k_s, k_g are the source and receiver horizontal wavenumber. Then, let define $b_1(k_g,k_s,\omega)$ which correspond to an uncollapsed f-k migration of effective incident plane-wave data as

$$b_1(k_g,k_s,\omega) = (-2iq_s)b_1(k_g,k_s,\omega) \tag{9}$$

where $q_s = sgn(\omega)\sqrt{(\frac{\omega}{c_0})^2 - k_s}$ is the source vertical wavenumber and c_0 the reference velocity. The second term in the algorithm is the leading-order attenuator b_3, which attenuates all the first-order internal multiples. The leading-order attenuator for a 2D earth is given by,

$$\begin{aligned} b_3(k_s,k_g,\omega) \quad &= \tfrac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} dk_1 \int_{-\infty}^{+\infty} dk_2 e^{-iq_1(z_g-z_s)} e^{-iq_2(z_g-z_s)} \\ &\times \int_{-\infty}^{+\infty} dz_1 b_1(k_g,k_1,z_1) e^{i(q_g+q_1)z_1} \\ &\times \int_{-\infty}^{z_1-\varepsilon} dz_2 b_1(k_1,k_2,z_2) e^{-i(q_1+q_2)z_2} \\ &\times \int_{z_2+\varepsilon}^{+\infty} dz_3 b_1(k_2,k_s,z_3) e^{i(q_2+q_s)z_3} \end{aligned} \tag{10}$$

where z_1, z_2 and z_3 are the pseudo-depths. ε is a small positive parameter chosen in order to make sure that $z_1 > z_2$ and $z_3 > z_2$ are satisfied.

Finally, using the input data and the leading-order attenuator of the first-order internal multiples, the data with the first-order internal multiples attenuated is given by

$$D(k_g,k_s,\omega) + D_3(k_g,k_s,\omega) \tag{11}$$

with $D_3(k_g,k_s,\omega) = (-2iq_s)^{-1} b_3(k_g,k_s,\omega)$.

COMPUTATIONAL COST SAVING METHOD : ANGLE CONSTRAINTS.

Terenghi et al. (2012) discuss about two angular quantities that can be used in order to reduce the computational cost of any algorithm defined in source and receiver transformed domain. The idea is to construct key-control parameters that allow to disregard some part of the calculus that is insignificant during the computation. In other words, use this key-paramters to optimize some intervals of calculus in the algorithm. The approach used is based on certain angular quantities in order to control the cost of the algorithm.

Stolt and Weglein (2012) define the image function wavenumber as a difference between the receiver and source-side wavenumbers

$$\vec{k_m} = \vec{k_g} - \vec{k_s} = (\vec{\kappa_g} - \vec{\kappa_g}, \vec{q_g} - \vec{q_s}) \tag{12}$$

With $\vec{\kappa_s}$ and $\vec{\kappa_s}$ the horizontal component of the source and receiver wavenumbers. These definitions allow the construc-

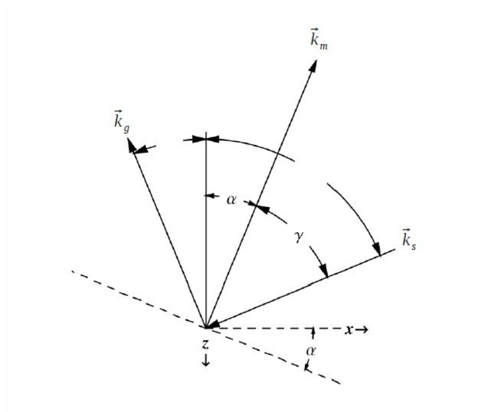

Figure 1: Plane waves at an interface in the subsurface. α is the angle between $\vec{k_m}$ and the vertical component. γ is the angle between $\vec{k_m}$ and $\vec{k_g}$ or $\vec{k_s}$. Figure from Terenghi et al. (2012).

tion of two angles α and γ (cf. Figure 1): α the dip angle corresponds to the angle between the surface and the horizontal component. γ the incident angle is the angle between the image function wavenumber and the source (or receiver) side wavenumber. Using simple trigonometry, α and γ can be related to the field quantities in the f-k domain:

$$\alpha = tan^{-1}\left(\frac{\sqrt{\vec{\kappa_m}.\vec{\kappa_m}}}{\mid q_g - q_s \mid}\right) \tag{13}$$

$$\gamma = \frac{1}{2}\left(-\frac{c_0^2}{\omega^2}(\vec{\kappa_g}.\vec{\kappa_s} + q_g q_s)\right) \tag{14}$$

The dependence of α and γ on the temporal frequency is carried by the occurrences of the vertical wavenumber q. Moreover, the relationship between α, γ and ω is monotonic. This means that at fixed values of $\vec{\kappa_s}$ and $\vec{\kappa_g}$ any given value of ω univocally identifies angles α and γ. Then, increasing the temporal frequencies in the data maps to decreasing values of the reflection dip and the aperture angle. At set value of $\vec{\kappa_s}$ and $\vec{\kappa_s}$, it is possible to conclude that any desired finite angle-domain interval maps to a similar finite frequency domain interval. This may be used in order to decrease the number of loop. Indeed, looking at the eqs (10), b_3 have - in 2D - two integrations over the wavenumber component. Therefore, it is possible to constrain the algorithm within a range of angular quantities,

$$\alpha_{min} \leq \alpha \leq \alpha_{max} \tag{15}$$

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$$\gamma_{min} \leq \gamma \leq \gamma_{max} \quad (16)$$

Using the α/γ and ω monotonic relationship, the total frequency interval can also be constrained,

$$max(\omega_\gamma^{min}, \omega_\alpha^{min}) \leq \omega \leq min(\omega_\gamma^{max}, \omega_\alpha^{max}) \quad (17)$$

Then, the reduction of the total frequency interval allows to reduce the interval of integration of b_3, which means reducing the number of loop.

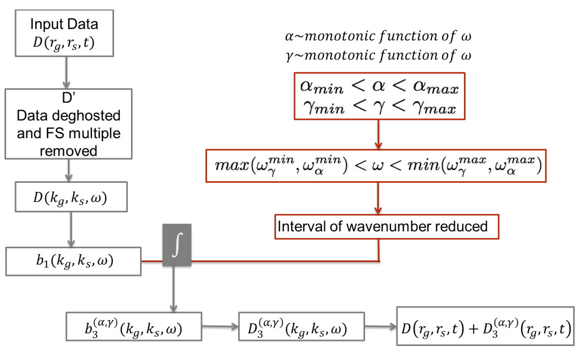

Figure 2: Process of the ISS internal multiple attenuation with angle constraints.

Figure 2 recapitulates in a graph the ISS internal multiple algorithm with angle constraints.
In the next section, a numerical analysis continues and illustrates the discussion, in which the efficiency and accuracy of the angle constraints method applied to the ISS internal multiple attenuation algorithm is discussed.

NUMERICAL ANALYSIS

The model considered in this numerical analysis is a three layer earth at depth : $z =$ 1000m, 1300m and 1700m. The source shot ($z = 910$ and $x = 6086$) is recorded by 928 receivers. In Figure 3 is the shot gather with primaries (green arrow) and internal multiples.

In the Figure 4, is the internal multiple prediction using the ISS internal multiple attenuation algorithm. All the first-order internal multiple are predicted. The model is in 1D; consequently just one angle (the incident angle γ) can be used as a key-control parameter. The analysis made in 1D for γ can be extended to α by analogy.
The Figure 5 illustrates the internal multiple prediction that uses angle constraints, as shown in Figure 2, for different γ_{max}. Also, for the same γ_{max} the percentage of time saved is listed in the Table 1.

γ_{max}	15°	20°	25°
Percentage time saved	67 %	57 %	50 %

Table 1: Time saved (in %) for the different γ_{max} studied.

A first interpretation would be that we do not need to compute for a full open γ-angle (90° degree by definition) to obtain an

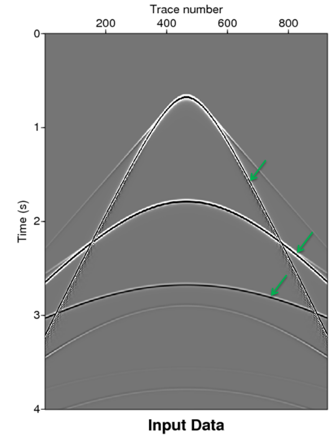

Figure 3: Shot gather recorded. The three primaries resulting from the three layers-reflectors are shown by the green arrows.

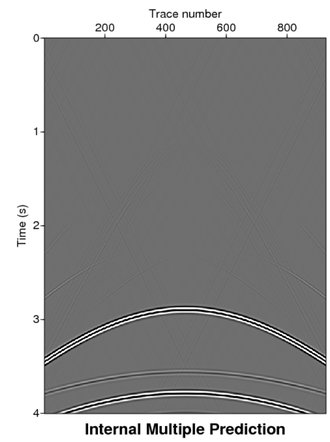

Figure 4: Prediction of all the first-order internal multiples.

accurate prediction of the internal multiples. Notice that a prediction with a full open angle corresponds to an internal multiple prediction without any angle constraints. Even so, with reduction to a certain angle (γ_{limite}), the prediction of the internal multiples is degraded.
For one trace number (750), is plotted in the Figure 6 the amplitude for different γ_{max} and compared with the amplitude for a full open γ-angle. In the Figure 5, the prediction of the internal multiples for $\gamma_{max} = 20°$ seems to be the same as $\gamma_{max} = 25°$ and Figure 4. If we look more precisely to the amplitude, we can notice that it had been affected. The amplitude for $\gamma_{max} = 20°$ do not overlap with the amplitude for $\gamma_{max} = 90°$ contrary to $\gamma_{max} = 25°$.
If we look at the shape (cf. Figure 7), the same interpretation can be made. For $\gamma_{max} = 25°$ the shape matches with an usual internal multiple prediction (full open γ-angle). Bellow this incident angle, the shape do not match which means that the prediction can not be considered accurate.

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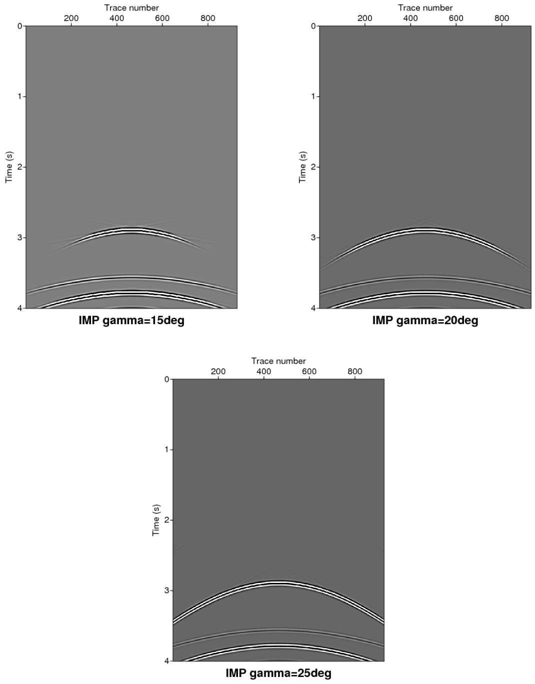

Figure 5: Internal multiple prediction for different angles of γ: $\gamma_{max} = 15^\circ$, $\gamma_{max} = 20^\circ$ and $\gamma_{max} = 25^\circ$.

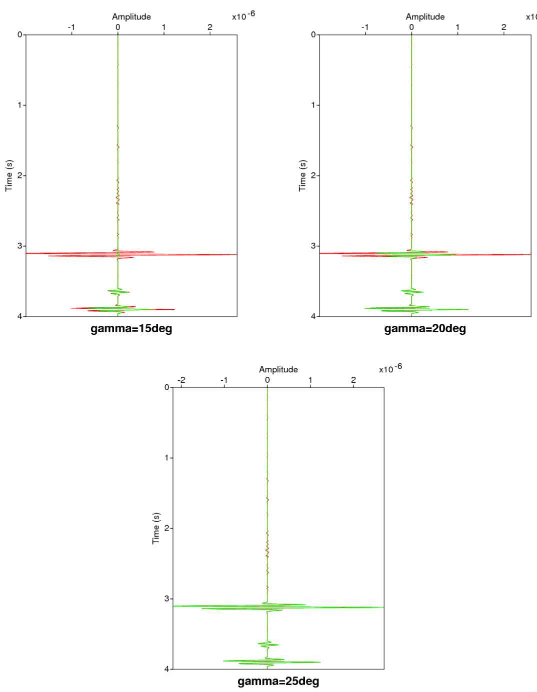

Figure 6: Comparaison of the amplitude for a full open γ-angle (red) and for different γ_{max} (green).

DISCUSION AND CONCLUSION

Terenghi et al. (2012) have introduced a time saving method:

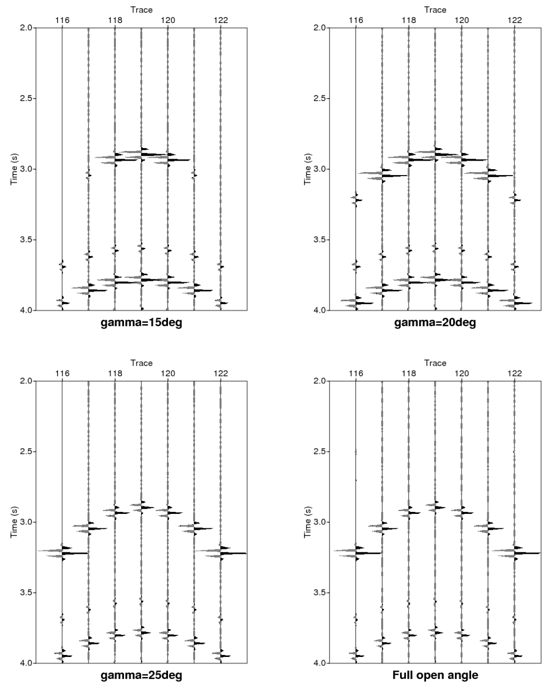

Figure 7: Wiggle plot for $\gamma_{max} = 15^\circ$, $\gamma_{max} = 20^\circ$, $\gamma_{max} = 25^\circ$ and full open γ-angle. Source at trace number 119.

the angle constraints. Looking at the procedure (cf. Figure 2), it is undeniable that applied to an algorithm defined in source and receiver transformed domain like the ISS internal multiple attenuation, this approach can reduce considerably the computational cost of the algorithm. Studying the impact of this key-control method in the algorithm, it appears that a compromise between the time saved and the accuracy of the internal multiple prediction has to be made. Indeed, above a certain ”angle limit” the internal multiple prediction stays accurate and precise. Below, the internal multiples are still predicted at the right time but with an approximate amplitude. Thus, the angle constraints is a trade-off tool between accuracy and cost of the algorithm. In other words, the ISS internal multiple algorithm will have its computational time reduced according to the degree of accuracy required by the user. The next step will be to identify this two angles using the input data in order to be able to define the constraint limits.

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EDITED REFERENCES

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